Ex°12 : Si $X \sim U[-1,1]$

Trouver la loi de $Y = e^{\frac{1}{X}}$.

$Y = \varphi(X) = \varphi(x) = e^{\frac{1}{x}}$.

- φ définie sur $\mathbb{R}^*$.
- φ de classe $\mathcal{C}^1$. $\varphi'(x) = -\frac{1}{x^2} e^{-\frac{1}{x}}$. $\varphi \downarrow$.

$\varphi(]-1,0[\cup]0,1]) =]e^{-1},0[\cup]e,+\infty[$

$\varphi(x) = y \Rightarrow x = \varphi^{-1}(y)$.

$\Rightarrow \frac{1}{x} = \ln y \Rightarrow x = \frac{1}{\ln y}$

- Si $y < 0 \Rightarrow F_X(y) = 0$.
- Si $y \in]0,e^{-1}[$, $F_Y(y) = P(\varphi^{-1}(y) \leq X \leq 0)$

$= F(0) - F(\varphi^{-1}(y))$

- Si $y \in]e^{-1},e[$. $F_Y(y) = P(X \leq 0) = \frac{1}{2}$.

Si $y \geq e$ $F_Y(y) = P(X \geq \varphi^{-1}(y)) = 1 - P_X(\varphi^{-1}(y))$

$f_Y(y) = -f_X\left(\frac{1}{\ln y}\right) \cdot \left(\frac{-1}{\ln^2 y} \cdot \frac{1}{y}\right)$

$f_X\left(\frac{1}{\ln y}\right) = \begin{cases} 1/2 & \text{si } \ln y > 1 \\ 0 & \text{si } \text{———} \end{cases}$

$f_Y(y) = \frac{1}{2y \ln^2 y} \mathbb{1}_{]0,e^{-1}[\cup]e,+\infty[}(y)$. page 8/10